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R.Q. Twiss

To cite this article: R.Q. Twiss (1969) Applications of Intensity Interferometry in Physics and Astronomy, Optica Acta: International Journal of Optics, 16:4, 423-451, DOI: [10.1080/713818198](https://doi.org/10.1080/713818198)

To link to this article: <https://doi.org/10.1080/713818198>



Published online: 11 Nov 2010.



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Applications of intensity interferometry in physics and astronomy†

R. Q. TWISS

Outstation of Royal Observatory of Edinburgh
Osservatorio Astronomico di Roma
00040, Monte Porzio, Catone, Italy

(Received 17 January 1969)

Abstract. Criteria as to when an intensity interferometer is competitive with a Michelson interferometer are established with the help of a simple example. There do not appear to be any cases in the field of frequency measurement where this is true. In radio astronomy it is argued that intensity interferometry might still have a role in angular measurements of the very smallest sources but it is clear that the main application still lies in visual astronomy in measurements in hot single stars and close spectroscopic binaries. A discussion is given as to how far the method could be pushed both from the technical as well as from the observational aspect. The triple correlation interferometer of Gamo is considered but because of its extremely low sensitivity it seems to have no application at least in visual astronomy and in this field the use of lasers is also rejected partly because of the various serious effects of atmospheric 'seeing', and partly because of the excessively tight tolerances on the performance of the very large mirrors needed to obtain adequate sensitivity. The possibility of laboratory applications also seem remote, in particular the idea that intensity interferometry could be used at x-ray wavelengths to measure the phase of scattering amplitudes is shown to be ruled out by inadequate sensitivity which could only be overcome by the development of coherent x-ray sources.

1. Introduction

The phenomenon of intensity interferometry can arise in any radiation process which exhibits wave-like properties. This includes the interaction of fermions as well as of bosons, and interference between acoustical as well as between Hertzian waves. However, by the terms of reference of this symposium, the present discussion will be confined entirely to the case of electromagnetic radiation.

This paper is not concerned with the basic principles of interferometry but, rather, with the applications, and when considering these it is clearly necessary to bear in mind the relative advantages and disadvantages of any alternative technique. At least for astronomical application, both at radio and at optical wavelengths, the alternative technique will be essentially some form of Michelson's stellar interferometer. A comparison of the two instruments is therefore called for in deciding, in any given case, whether an intensity interferometer can usefully be employed. As a background to such comparisons, which naturally depend upon the details of the specific problem, we give, in the next section, a rather general discussion with the aim of isolating a few basic conditions, some of which at least would have to apply before one would seriously consider using an intensity interferometer.

† Paper presented at the Symposium on Applications of Coherent Light, Florence, invited 1968.

2. Basic principles of intensity interferometry

The fundamental theory appropriate to optical wavelengths has been discussed in considerable detail in a series of four papers [1-4] designated, in what follows, by the Roman numerals I-IV. It may be helpful here to supplement this work by considering a particularly simple example of intensity interferometry, less specifically oriented to the optical case, from which it will be possible, by more or less heuristic arguments, to lay bare the main features of an intensity interferometer. The reader interested in a more complete and rigorous discussion is referred to I-IV.

Let us suppose that there are two simple harmonic transmitters out in space, so far distant from the Earth that the incident radiation from either transmitter is effectively a plane wave. An idealized intensity interferometer would then consist of two square-law detectors, of limited aperture, spaced a distance d apart on the ground, the low frequency a.c. outputs of which are brought together by cables and cross-correlated in a linear multiplier. One can then use the instrument in two separate ways by observing how the correlation varies:

- (a) with baseline d , as in a *spatial* intensity interferometer, to obtain information about the apparent angular spacing of the transmitters;
- (b) with differential time delay τ_0 in the two interferometer cables, as in a *temporal* intensity interferometer, to obtain information as to the frequency separation of the two transmitters.

Specifically let us assume that the transmitters radiate simple harmonic waves of amplitudes E_1 and E_2 and frequencies ν_1 and ν_2 , respectively, where

$$\frac{\nu_1 - \nu_2}{\nu_0} \ll 1, \quad \nu_0 = \frac{\nu_1 + \nu_2}{2}, \quad (2.2)$$

and that the phases of the signals at O, the mid point of the baseline joining the two detectors are:

$$\phi_1 \quad \text{and} \quad \phi_2 \quad \text{respectively.}$$

Now let us take a Cartesian coordinate system with O as origin, with the z axis intersecting the line joining the two transmitters and with the x axis in the plane containing the two transmitters. It is then easy to show that the beat notes in the two detector outputs are proportional to:

$$AE_1E_2 \cos \left[2\pi\nu_1 - \nu_2 t - \phi_1 - \phi_2 \pm \frac{\pi d \cos \psi}{c} (\nu_1 \sin \theta_1 - \nu_2 \sin \theta_2) \right], \quad (2.2)$$

where ψ is the angle between the x axis and the projection of the baseline onto the x - y plane, A is the area of the detector, θ_1, θ_2 are the angles between the z axis and the direction of the transmitters.

In the important case which would normally hold in any practical application, in which

$$\frac{\sin \theta_1 - \theta_2}{\theta_1 - \theta_2} \simeq 1 \quad (2.3)$$

and

$$\frac{\nu_1 - \nu_2 d \theta_1}{c}, \quad \frac{\nu_1 - \nu_2 d \theta_2}{c} \ll 1, \quad (2.4)$$

this expression for the beat note output of the detectors simplifies to:

$$AE_1E_2 \cos \left[2\pi \overline{\nu_1 - \nu_2} t - \overline{\phi_1 - \phi_2} \pm \frac{\pi d}{\lambda_0} \overline{\theta_1 - \theta_2} \cos \psi \cos \frac{\overline{\theta_1 + \theta_2}}{2} \right], \quad (2.5)$$

where

$$\lambda_0 = c/\nu_0 \quad (2.6)$$

is the mean wavelength of the incident radiation.

If now these two outputs be first delayed by a time τ_0 with respect to each other and then be correlated together in a linear multiplier the time averaged output of the latter is proportional to:

$$A^2 I_1 I_2 \cos \left[\frac{2\pi d}{\lambda_0} \overline{\theta_1 - \theta_2} \cos \psi \cos \frac{\overline{\theta_1 + \theta_2}}{2} - 2\pi \overline{\nu_1 + \nu_2} \tau_0 \right], \quad (2.7)$$

where

$$I_1^2 = \frac{E_1^2}{2}, \quad I_2^2 = \frac{E_2^2}{2} \quad (2.8)$$

are the intensities of the two transmitters at the detectors.

This equation was derived on the assumption that

$$\frac{\pi D}{\lambda_0} (\theta_1 - \theta_2) \ll 1, \quad (2.9)$$

where D is a linear dimension of the detector aperture. To see what happens when this is not valid, consider the simple case of a rectangular detector with area $A = XY$. Then as one moves across the detector parallel to the x axis, that is to the line joining the transmitters, it is clear from (2.5) that the phase of the beat note will vary since this phase, while independent of y depends explicitly upon x . The total detector output is then proportional, not to A , but to:

$$A \frac{\sin (\pi X \overline{\theta_1 - \theta_2} / \lambda_0)}{\pi X \overline{\theta_1 - \theta_2} / \lambda_0}.$$

It follows that the output of an intensity interferometer for large A increases not as A^2 but only as A , a fact of great practical importance which sets a lower limit to the brightness temperature of any source that can be used in an intensity interferometer experiment.

3. Comparison of intensity interferometer with Michelson's interferometer

To begin with let us consider the *spatial* interferometer case in which one looks for angular information. From (2.7), to do this, one must keep τ_0 constant or, preferably, zero, and vary the projected baseline d_0 , where

$$d_0 = d \cos \psi \cos \frac{\theta_1 + \theta_2}{2}. \quad (3.1)$$

In this case the output of the intensity interferometer will vary sinusoidally with d_0 , the separation between successive maxima and minima being given by:

$$\Delta d_0 = \frac{\lambda_0}{2\theta_1 - \theta_2}. \quad (3.2)$$

In the *temporal* interferometer one would keep d_0 constant, or preferably zero and would vary τ_0 when the separation between successive maxima and minima would be given by:

$$\Delta\tau_0 = \frac{1}{2(\nu_1 - \nu_2)}. \quad (3.3)$$

Accordingly, in this simple example of two transmitters, it may be seen that the resolving power of the intensity interferometer is exactly the same as that of the classical Michelson interferometer the output of which will, in the present instance, contain a term equal to:

$$I_1 \cos 2\pi \left(\frac{d}{\lambda_1} \cos \psi \sin \theta_1 - \nu_1 \tau_0 \right) + I_2 \cos 2\pi \left(\frac{d}{\lambda_2} \cos \psi \sin \theta_2 - \nu_2 \tau_0 \right) \quad (3.4)$$

which will oscillate rapidly with d in the familiar fringe pattern. The information as to the angular separation of the two transmitters is conveyed by the sinusoidal modulation of the amplitude of this fringe pattern, the so-called 'fringe visibility', the separation between successive maxima and minima of the fringe visibility being given by:

$$\Delta d_0 = \frac{\lambda_0}{2(\theta_1 - \theta_2)} \quad (3.5)$$

in the case

$$\lambda_1 \approx \lambda_2, \quad \theta_1, \theta_2 \ll 1. \quad (3.6)$$

Similarly as τ_0 is varied, the separation between successive maxima and minima of the fringe visibility is given by:

$$\Delta\tau_0 = \frac{1}{2(\nu_1 - \nu_2)}. \quad (3.7)$$

The equal resolving power of the two instruments for this simple case of two monochromatic transmitters is established at once by comparing equation (3.2) with (3.5) and equation (3.3) with (3.7). However, this is not the only point of comparison by any means, as may be seen by asking what other information can be learned about the source. With the intensity interferometer, the answer is very little. Thus in a frequency type measurement one can only measure the frequency difference of the two transmitters, one knows nothing about the central frequency. As long as one measures $A(I_1 + I_2)$ the d.c. output of a detector, one can measure the relative strength of the two transmitters but one does not know which is the stronger. With the Michelson in the other hand one does measure the mid-band frequency and one *can* decide whether the lower or the higher frequency transmitter is the stronger. Similarly in a spatial type measurement the intensity interferometer measures only the angle between the two transmitters as along as

$$\cos \frac{\theta_1 + \theta_2}{2} \approx 1, \quad (3.8)$$

which will always be true in a practical application. Subject to equation (3.8) one has no information as to the absolute position of the two transmitters in space nor, of course, does one know whether the left-hand or right-hand one is

the stronger. With the Michelson interferometer one *can* determine the orientation of the transmitters by identifying the central fringe, and one *can* decide which source is the stronger.

So far we have considered the simple case of two transmitters. When there are N transmitters the Michelson interferometer will yield N simple harmonic terms in the Fourier analysis of the interferometer record, but in the intensity interferometer case there will be $N(N-1)/2$ beat notes to identify. Not only will the computations be much more complex but there will always be some ambiguity in the interpretation and, because of the non-linear nature of the equations, the details of the distribution are much more easily lost than in the Michelson case. A similar argument applies to the N -fold spatial distribution.

If this were the whole story, there would clearly be little point in an intensity interferometer. However, as is so often the case, the additional information given by the Michelson interferometer is not purchased for nothing. In the present instance the price is that the Michelson interferometer is much more critical mechanically, while its operation can be much more seriously affected by disturbances in the medium through which the incident radiation is propagated.

The reason for the independence of an intensity interferometer to such disturbances can be seen at once from equation (2.2), which shows that the phase of the beat note in the output of a detector depends only upon the *relative* phase of the signals from the transmitters, and from equation (2.7) which shows that differential time delays around the interferometer only appear as a phase change in the *beat note*. It follows that fluctuating time delays in either path of the interferometer, whether caused by mechanical vibration in the mount or by fluctuations and irregularities in the medium, can have no significant effect on the reading of an intensity interferometer as long as the amplitude of such time delays be small compared with $1/b_v$, where b_v is the post detector bandwidth equal, in the present case, to $\nu_2 - \nu_1$. Furthermore, it does not matter if signals reaching one part of the detector from one transmitter differ significantly in phase from those reaching another part as long as this phase difference is the same for both transmitters. In fact as far as the signal is concerned we could form the detector in one path of the intensity interferometer by a whole set of quite separate detectors, the outputs of which were then added together, as long only as the time delays between these different detectors were equal to within a time error small compared with $1/b_v$.

In the case of the Michelson interferometer, on the other hand, things are very much more critical since fluctuations in the relative time delay will lead to significant fluctuations in the fringe pattern, unless the differential delays are small compared with $1/\nu_2$, while the fringe pattern due to radiation incident at one point of the aperture will only add coherently to the fringe pattern due to radiation incident at another point if the *signal* as opposed to the *beat note* phase is equal all over the aperture. In other words problems arise in a Michelson interferometer not only if one does not maintain stable phase at opposite ends of the baseline, but also if one does not maintain a stable uniform phase over the aperture.

Unfortunately in many cases this advantage is more than offset by the fundamental drawback to the use of an intensity interferometer which is its inherently low signal to noise ratio due essentially to the non-linear element inside the interferometer proper.

Thus let us consider the practical case where, instead of two isolated transmitters, one has an extended source of thermal emitters. If a measurement with an ideal Michelson interferometer has a signal to noise ratio given by:

$$\frac{S}{N}(T_0)_M = C_M \rho \sqrt{BT_0}, \quad (3.9)$$

then the signal to noise ratio in an equivalent measurement with an intensity interferometer would be given by:

$$\frac{S}{N}(T_0)_I = C_I \rho^2 \sqrt{bT_0}, \quad (3.10)$$

where C_M and C_I are numerical constants of the order of unity which depend upon the specific experimental set-up, B and b are the predetector and post-detector bandwidths respectively and T_0 is the observation time. If $\rho \ll 1$, as is often the case, then very long integration times may be needed to obtain adequate accuracy in observations with an intensity interferometer.

This comparison is of course far from complete but it does serve to bring out that the time to consider using an intensity interferometer is when for some reason or other the use of a Michelson interferometer is suspect. In practice this is likely to occur either at very high resolving powers where the mechanical or electro-optical problems in a Michelson interferometer might get too severe, or when disturbances in the medium are such as seriously to degrade the performance of the more sensitive instrument. In any case it must be emphasized that an intensity interferometer is not a survey instrument, because its sensitivity is too low, it is not for examining complex sources either for angular or for frequency information, nor is it for getting detailed or fine structure information about an isolated source, because it measures the square of the amplitude of the Fourier transform rather than the transform itself. On the contrary its use is likely to be largely confined to determining the main features of simple distributions such as the angular size of a simple source or the angular separation of two sources or the width of an isolated but very narrow spectral emission line.

4. Applications to radio astronomy

The first application of intensity interferometry, which was in radio astronomy, was directly inspired by the need for high angular resolution [5]. At the time it was thought that the 'point' radio sources, as they were then called, might well be of stellar dimensions and to resolve a true radio star at the metre wavelengths then favoured by radio astronomers needed baselines comparable with, or possibly larger than, the diameter of the Earth.

However, as is well known, the typical radio source is of nebular rather than stellar dimensions and to measure the brightness temperature and even the structure of these does not call for great resolving power. For this reason no use was made of intensity interferometry, at least at radio wavelengths, for many years after the initial observations [5, 6]. It should be noted however that these were the first to fully resolve a 'point' source and to show the double component structure of Cygnus A*, so characteristic of many of the radio sources of extragalactic nature and high absolute luminosity.

4.1. *Radio intensity interferometry at very high resolution*

With the discovery of quasars, of OH emission features of very small apparent size, and, above all, of pulsars, the need for very high resolution at radio-frequency wavelength has become real and has made it once again logical to look at the possibility of using an intensity interferometer. Indeed one has already been used in observations of 3 C 273 B with a baseline of 9 000 000 wavelengths [7]. At these baselines, and at the even greater ones needed to resolve the sources completely, Michelson interferometers, using radio links to transmit phase stable signals, meet severe problems, but successful measurements have been made out to 10^8 wavelengths using ultra high stability local oscillators combined with wide-band magnetic tape recorders [8]. One could also use the latter to record the post-detector signals in a very high resolution intensity interferometer, as indeed was envisaged when the technique was first mooted, but the very high stability local oscillators would no longer be needed and, in other ways as well, the intensity interferometer, would be much simpler technically. In what follows we shall compare the performance of the two types of instrument assuming that the pre-detector bandwidth of the Michelson type interferometer is equal to the post-detector bandwidth of the intensity interferometer and that both are set either by the emission line width, as in the case of the OH sources, or by the limitations of the tape recorder and by the problems of correlating the two outputs with a time delay error small compared with the inverse bandwidth which are common to both techniques. Before doing this one must first give a defence against the argument that it is now too late to worry about intensity interferometry when very long baseline interferometers have already been made to work satisfactorily with the more sensitive Michelson instrument. Admittedly there is a lot in this objection but there are two points to be made against it which, in the writer's view, have enough weight at least to justify serious consideration of an intensity interferometer. In the first place there is always much to be said for confirming the results found by one technique against those found by a quite different type of measurement and this is especially true in measurements on the smallest sources because, in interferometry, all errors have the effect of reducing the fringe visibility and thus suggesting that the source is resolved. In the second place yet smaller sources may one day be found too small to be resolved by any earth-bound baseline, indeed it seems quite possible that the pulsars may be in this category, especially if they are really associated with neutron stars a few kilometres in radius. If one is to consider extra terrestrial observation at all the attractions of the simplest possible technique become that much the more real.

In radio astronomy one cannot, of course, use just a detector as in the simple example of §§ 2 or 3, but one has to have a large aerial feeding power into a radio receiver, either with or without an i.f. amplifier, which in turn feeds a square-law-detector. Unlike the optical case, in which the detector is directly at the focus of the mirror, one cannot use a crude mirror of poor electrical quality or the aerial gain in the main lobe would deteriorate seriously with catastrophic effects upon the signal to noise ratio. It is true however that a strong signal in the side lobes is much less serious in the intensity interferometer where the output is proportional to the product of the aerial power gains than in the Michelson case where it is proportional to the product of the corresponding voltage gains.

When the source can be represented as a spatial distribution of incoherent radiators, as can always be done in an astronomical application, whether at optical

or radio wavelengths and whatever the local radiation mechanism may be, one can use the theory given in I–IV to show that the intensity interferometer measures the quantity :

$$\Gamma^2(d, \psi_0),$$

where $\Gamma(d, \psi_0)$, the fringe visibility, is the amplitude of the Fourier transform of the intensity distribution over the equivalent line source projected on a baseline spacing d and orientation ψ_0 .

As long as the source is unresolved by an individual aerial, as certainly applies at radio wavelengths to any small source, the signal to noise ratio in an intensity interferometer measurement is given, under certain conditions to be described by :

$$\frac{S}{N}(T_0)_I = \left(\frac{P_S \sqrt{(A_1 A_2)}}{P_N + P_S \sqrt{(A_1 A_2)}} \Gamma(d, \psi_0) \right)^2 \sqrt{(2b_v T_0)}, \quad (4.1)$$

where

P_S is the signal power per unit frequency bandwidth per unit area received from the source,

P_N is the effective noise power comprising both receiver and galactic noise,

A_1, A_2 are the effective aerial apertures,

b_v is the post-detector bandwidth,

T_0 is the total observation time.

In deriving this result it was assumed first of all that the r.f. and i.f. bandwidths were much larger than the post detector bandwidth and that they were of an ideal square-topped shape. This would not necessarily apply in measurements in the OH sources because of the narrowness of the emission lines, in this case it would be necessary to match the r.f. and post-detector responses and the signal to noise might be reduced by a further factor of $1/\sqrt{2}$. Secondly, the expression is only rigorously valid when $\Gamma(d, \psi_0)$ is small, so that the correlation between the signal fluctuation in the two channels is also small. When this is not so a much more complex expression must be used but fortunately the error even then in using equation (4.1) is small as long as† :

$$P_S \sqrt{A_1 A_2} \ll P_N.$$

For the Michelson interferometer the corresponding expression under similar assumptions is given by :

$$\frac{S}{N}(T_0)_M = \frac{P_S \sqrt{(A_1 A_2)}}{P_N + P_S \sqrt{(A_1 A_2)}} \Gamma(d, \psi_0) \sqrt{(B_M T_0)}, \quad (4.2)$$

where B_M is the pre-detector bandwidth. In the present case this is to be taken equal to b_v , which, in turn, is not greater than the bandwidth of the magnetic tape recorder. Under these conditions the relative advantage in sensitivity of a Michelson interferometer is very much less than, for example, in the optical case where the predetector bandwidth may be of the order of 10^{12} – 10^{13} cycles, at

† The chief reason that we have allowed for the contribution of the signal fluctuation to the noise is to emphasize that the signal to noise ratio cannot be improved indefinitely even on a 'point' source, simply by increasing the aerial aperture, a fact that is relevant to give a concrete example, to observations on the intense OH source.

least 10^4 times the highest practical post-detector bandwidth. Indeed the two instruments have virtually the same sensitivity when $P_S \approx P_N$, as will be the case on the brightest OH sources, as long as the aerial aperture is of the order of 30 m or more. More generally if G be the ratio of the luminosities of the faintest sources that can be detected with the two techniques:

$$G = 8.5 \sqrt{\left(\frac{R_0}{10}\right) \left(\frac{BT_0}{10^6}\right)^{1/4}}, \quad (4.3)$$

where R_0 is the signal to noise ratio to which one wishes to work. For $R=10$, $B=10^4$ Hz, $T_0=10^2$ sec this luminosity ratio is equal to a little less than 2.5 stellar magnitudes and in practice this comparison is likely to be over favourable to the Michelson interferometer for which the technical problems in reaching the ideal sensitivity are enormously greater than in the case of the intensity interferometer.

Thus in the first place the relative drift, during an observation, between the frequencies of the two local oscillations at opposite ends of a baseline must not lead to a relative phase drift much larger than $2\pi/10$ if significant loss of correlation is to be avoided in a Michelson interferometer. At an r.f. frequency of 10^9 Hz and an observation time of 10^3 sec this implies a short term stability of 1 in 10^{13} , this is also the accuracy to which the oscillation frequency difference must be known and this allowable error decreases linearly with the inverse observation time. In the intensity interferometer it is only necessary that the stability be good compared with the fractional pre-detector bandwidth. If this be 10^{-3} a frequency stability of 1 in 10^5 independent of the observing time is quite adequate and even in the case of the OH sources where the predetector bandwidth is inherently small one needs at worst, to demand a local oscillator stability of 1 in 10^8 .

In the second place because there is no fringe pattern the output of an intensity interferometer is a d.c. signal and there is no problem with present day techniques in integration times of many hours. Indeed this has already been achieved in the much more complex case of an optical intensity interferometer in which the postdetector bandwidth was of the order of 10^8 Hz. In the Michelson interferometer however one has a fringe pattern the period of which is a function of the hour angle and declination of the source as well as of the orientation and length of the baseline. With an East–West baseline 10^8 wavelengths long there will be nearly 10^6 fringes in a 1000 sec observation period for a source near to transit. The problems in measuring the associated ‘fringe visibility’ in this case with the theoretical optimum signal to noise ratio are not negligible and get progressively harder as the observation time is increased.

Finally there is the prospect of improving the signal to noise ratio of the intensity interferometer by multi-channel techniques. The most effective way to do this is to add constructively the low frequency detected outputs from a number of aerials. With M equal aerials at each end of the baseline this would improve the signal to noise ratio M times compared with the improvement of M^2 that would arise with a single aerial of M times the collecting area. For this multi-aerial technique to work it is necessary only that the relative time delays be small compared with the inverse post-detector band. If this be 10^4 Hz the relative time delays should not be more than $10 \mu\text{sec}$ and in this case only one tape recorder channel need be used at either end of the baseline. If one is only to use one aerial one can use N channels each with its own detector. In this case however the detector

outputs are not mutually coherent, a separate magnetic tape channel is needed for each of the N outputs and the improvement in signal to noise ratio is proportional only to $N^{1/2}$.

5. Applications to optical astronomy

In the last section we argued that the intensity interferometer might still have a useful role to play in radio astronomy even though a very minor one compared with the Michelson interferometer. However, at optical wavelengths this comparison must be reversed at least at the present time, because no results of any kind have been reported of measurements with a Michelson stellar interferometer on the hot early type stars, the field which has seen the main application of intensity interferometry. Conceivably this may change in the future but it would take us too far from the subject of the present paper to discuss how the early measurements of cool stars by Michelson and Pease [9] with the 20 ft stellar interferometer might be extended to much greater baselines. Accordingly in what follows we shall confine ourselves to the intensity interferometer and shall review the possible applications discussed earlier [3] in III in the light of present knowledge and of the observational results already published and we shall also discuss possible future developments both from the observational and from the technical and instrumental angle.

At optical wavelengths there are three fundamental reasons for using an intensity interferometer in place of a Michelson interferometer two of which do not apply at all at radio wavelengths.

(i) Very great resolving power is needed to measure even the closest of the hot stars. Thus fully to resolve β Crucis, the brightest Bo star in the sky, requires a baseline of 3.6×10^8 wavelengths [10] that is 150 m at 4300 Å. The problems in using a Michelson interferometer of this size are formidable.

(ii) At optical as opposed to radio wavelengths the Michelson interferometer is seriously affected by the atmospheric turbulence which produces the effect known as 'seeing'. This is especially true when the visual observer is replaced by photoelectric detectors and automatic data processing, as has to be done if one hopes to get accurate measurements of 'fringe visibility' and to improve the signal to noise ratio by long integration periods. The intensity interferometer, on the other hand, is virtually unaffected by 'seeing'. This statement is supported by observational evidence from the Narrabri interferometer [11] and also by the detailed theoretical discussion given [3] in III, which is essentially an expansion of the argument outlined in § 3 above.

(iii) In a Michelson stellar interferometer high quality optics and elaborate image stabilization techniques are needed if one is to obtain accurate automatic measurements of 'fringe visibility'. In the optical intensity interferometer, on the other hand, unlike the radio case, one can use comparatively crude 'light bucket' mirrors without any significant loss of either accuracy or sensitivity as long as the mirror is good enough to ensure that all the starlight falls on the cathode of the photomultiplier detector. The point is that the photomultiplier combines a true quantum detector in its photocathode with an electron multiplier chain of so high an internal gain that the noise in the associated intensity interferometer is determined almost entirely by the total incident light flux. Under these conditions one can also replace a single large crude mirror by an array of N equal

small mirrors. These can all be focused onto a single photomultiplier as in the Narrabri interferometer [11] or, alternatively, the light incident on each component mirror can be focused onto its own separate photomultiplier. As long as the differential time delays are equalized with adequate accuracy one can add coherently the signal terms in the low frequency outputs of the N separate photomultiplier with, ideally, the same signal to noise ratio as if all the light had been focused onto a single photomultiplier and with N times the ratio which one would get by using just one of the component mirrors.

It is this capacity to use crude, compound or multiple mirrors which is perhaps the most important distinction between the radio and the optical case and it is the essential factor which makes it possible to apply intensity interferometry to optical astronomy at an acceptable cost.

5.1. Signal to noise ratio

Even more than in the radio case the use of an optical intensity interferometer is limited by its sensitivity, so any discussion of the possible application of the technique must be based on the signal to noise ratio that can be reached in an observation time T_0 with a given set of instrument parameters.

For the case of narrow band unpolarized light with mid-band frequency ν_0 a general expression for the signal to noise ratio $S/N(T_0)$ is given [2] in equation (4.1) of II and this is quoted in equation (5.1) below with one or two minor changes in symbolism and one minor simplification in that we do not here allow for time dependent changes in atmospheric extinction or in background stray light. On this basis we have that

$$\frac{S}{N}(T_0) = A_1 A_2 \rho(\nu_0)^{\sec z} (\alpha \nu_0) n_0(\nu_0) \Delta(\nu_0) \Gamma^2(\nu_0, d) \sqrt{\left(\frac{b_\nu T_0}{2\eta}\right)} \Sigma, \quad (5.1)$$

where

- $\alpha(\nu_0)$ the geometric mean of the quantum efficiencies of the photomultiplier is equal to the product of the cathode quantum efficiency and the anode collector efficiency,
- $n_0(\nu_0)$ is the number of photons $\text{m}^{-2}\text{s}^{-1}(\text{c/s})^{-1}$ normally incident from the star on the top of the atmosphere at frequency ν_0 ,
- $\rho(\nu_0)$ is the atmospheric transparency at vertical incidence.
- z is the zenith angle of the star,
- A_1, A_2 are the effective collecting areas of the mirrors,
- b_ν is the cross-correlation post-detector bandwidth,
- η is the normalized post-detector spectral density [2] defined in II,
- $\Delta(\nu_0)$ is the partial coherence factor, never greater than unity, which defines the reduction in correlation at zero baseline when the star is so large as to be partially resolved by the individual mirrors.

The quantity Σ represents the loss of signal to noise ratio below the ideal for an equipment in which the main parameters such as quantum efficiency, mirror

aperture and post-detector bandwidth are given. It can be broken down into separate terms as follows:

$$\Sigma = \frac{\epsilon\gamma(\nu_0)\sigma}{(1+a)(1+\delta)\kappa}, \quad (5.2)$$

where

- $1+a$ is the ratio of the total light flux at each photocathode from the star plus stray light to the flux from the star alone,
- $1+\delta$ represents the excess noise produced in the electronic equipment including the linear multiplier,
- $\gamma(\nu_0)$ is the optical transmission efficiency of the interferometer up to the photo-cathode,
- $1-\epsilon$ is the combined loss of correlation in the optical and electronic equipment due, among other things, to differential time delays around the two channels.,
- σ is the normalized spectral density [2] of the filtered light incident on the photocathode defined in II,
- κ is the excess noise introduced by the process of photomultiplication. In III it was taken as $\mu/\mu-1$, where μ is the multiplication of the first dynode but a more realistic semi-empirical value is given by:

$$k = \frac{\mu}{\mu-1} \left(1 + \frac{C}{\mu}\right),$$

where C is a numerical constant with the typical value of 0.6.

As discussed [3] in III, $\Gamma^2(d, \psi_0)$ is not exactly equal to the square of the amplitude of the 'fringe visibility' when $\Delta(\nu_0)$ the partial coherence factor is measurably less than one, but the correction, which can always be applied in the computation, is small, and does not affect the present analysis.

5.2. The Narrabri interferometer [10, 11]

The discussion given [3] in III of the stars that could be measured with an intensity interferometer of given sensitivity was of necessity based entirely upon theory because there was then no observational experience to draw upon. This part of the earlier paper has now been quite superseded by the work of Hanbury Brown *et al.* with the intensity interferometer at Narrabri [10, 11], especially on such questions as to the minimum signal to noise ratio in one hour that is operationally significant. In any case it is clearly necessary to base oneself on these experimental results before one can hope to assess future developments.

The main parameters of the Narrabri interferometer [11] are given numerically by the relations:

$$\alpha = 0.20, \quad (A_1 A_2)^{1/2} = 29.5 \text{ m}^2, \quad b_v/\eta = 95 \text{ Mc/sec.} \quad (5.4)$$

A number of light filters were used at different times but the final results were taken [10] with

$$\lambda_0 = 4430 \text{ \AA} \quad \text{or} \quad \nu_0 = 6.77 \times 10^{14} \text{ Hz}, \quad (5.5)$$

when one may assume

$$\rho(\nu_0) = 0.69 \quad \text{and} \quad n(\nu_0) = 9 \times 10^{-5} \text{ photon m}^{-2} \text{ s}^{-1} (\text{c/s})^{-1} \quad (5.6)$$

as reasonable approximate values for the night sky transparency and incident photon flux respectively.

Finally we have calculated that the loss factor Σ was given by:

$$\Sigma = 0.174, \quad (5.7)$$

made up as follows†:

$$\left. \begin{aligned} \epsilon &= 0.9, & \sigma &= 0.84, \\ 1+a &= 1.01, & \gamma(\nu_0) &= 0.42, \\ 1+\delta &= 1.1, & \kappa &= 1.62. \end{aligned} \right\} \quad (5.8)$$

With these parameters the interferometer gave a signal to noise ratio of 24.3 to 1 in one hour when observing a zero magnitude star in the zenith with a very short baseline [11]. For hot stars the limiting apparent stellar magnitude on which observation could be made was +2.0, so that the limiting signal to noise ratio is one hour was just under 4 to 1. This may be compared with the preliminary value of 3 to 1 that was assumed [3] in III. At limiting magnitude with 50 hour's observation it was possible to measure the angular diameter of a hot star with an accuracy of ± 7.5 per cent [11]. This type of measurement on isolated hot stars is not the only one that can be carried out with an optical interferometer, as we shall see later, but it will serve for the present as a standard by which to judge how much we might hope to improve on the limit to 2nd magnitude stars. The astronomical possibilities of improved sensitivity will be discussed later.

5.3. Means of increasing sensitivity

5.3.1. Larger mirrors

The most obvious way to reach fainter stars is to increase the size of the interferometer mirrors since the sensitivity depends linearly upon the mirror area. The maximum reachable aperture is probably set more by financial than by engineering considerations but it should be possible to build mirrors comparable in size to the movable 90 ft paraboloid reflectors of the radio interferometer of the out-station of the California Institute of Technology at Owens Valley and gain an increase in sensitivity of about 3.25 magnitudes. Indeed, in many ways the tolerances of the radio mirrors are quite a bit tighter than in the optical case.

† All the numerical values in equation (5.8) were taken from Hanbury Brown *et al.* [10], except for the value of κ for the excess photomultiplier noise which they take to be 4/3 corresponding to a value of μ , the first dynode multiplication factor, of 4. This is almost certainly too low. From equation (5.3) above we should expect κ to be equal to 1.53 and there was definite evidence that the experimental signal to noise ratios were less than the theoretical when κ was taken as 1.37. Exact agreement is obtained for $\kappa = 1.62$ or in equation (5.8) above. One does not wish to belabour this because, as Hanbury Brown *et al.* [10], point out, the theoretical values for the signal to noise ratio in the Narrabri interferometer are not too certain but the point is that recent advances in photomultiplier technology have made it possible greatly to increase μ with a corresponding decrease in κ and it is important not to underestimate the potential improvement that this will bring.

For one thing in the latter, one could use N separate mirrors† each with its own detector and since one is always working on bright stars there is ample light to servo control each detector separately, thus lowering the mechanical requirements of the mount.

Secondly the radio telescope figure has to be good for the highest r.f. frequency at which it is to observe. The requirements for the optical intensity interferometer is at least ten times easier since in this case the criterion is set by the optical post-detector bandwidth.

5.3.2. *Simultaneous observations with short and long baselines*

In order to minimize the effects of systematic error and also to see whether the star under observation was a binary it proved necessary in the Narrabri observations to interleave observations at short and long baselines. Ideally these two observations should be simultaneous nor is it necessary or desirable that one should use three aerials. Instead, the light from one or both mirrors could be split into two either by a beam splitter near the focus of a single mirror or by interleaving component mirrors in the multi-mirror design. If this be done, at the price of duplicating the electronic equipment, one would not only minimize the possibility of error but would also improve the limiting sensitivity by nearly 0.5 magnitudes.

5.3.3. *Improved light detectors*

Three parameters of the detector are relevant in the context of improved performance in an intensity interferometer, the internal signal to noise ratio, the overall quantum efficiency and the bandwidth. The first of these depends upon the multiplication factor of the first dynode and, as we noted above, high gain dynodes have recently become available with stage gains as high as 30 which should reduce the excess phototube noise to a factor of ≈ 1.05 , a gain in sensitivity of 0.5 magnitudes. As for the quantum efficiency, this has been creeping steadily upwards over the last few decades and in the laboratory at least efficiencies as high as 32 per cent in the blue have been obtained in selected tubes. Eventually one can hope that such tubes will become available commercially with an associated increase of 0.5 magnitudes in sensitivity‡.

Finally with cross-field multiplication the possibility of an order of magnitude of increase in bandwidth is now real and in the future the post-detector bandwidth may be well set more by the correlator and by the operational problems of very large bandwidths than by the photomultiplier itself, as will now be discussed.

† This is not the only advantage of multiple mirrors. For a given f ratio the focal length of a composite mirror is shorter as $N^{-1/2}$, as is the diameter of the stellar image, an important consideration with very high bandwidth light detectors. In the same way, for a given size of filter optics, the magnification of the angular spread of the starlight due to mirror imperfections is also decreased as $N^{-1/2}$. Finally with composite mirrors large enough to observe faint stars one can use a single component mirror to observe bright stars without the need to mask the aperture, if the star is significantly resolved by the full mirror.

‡ This is of course well down on what can be had from good photodiodes in which the efficiency can reach nearly 100 per cent. However, at the present time there seems small prospect of getting the high internal gains of the order of 10^5 which are needed if the noise in the subsequent amplifiers and correlator is to be small compared with the photodetector shotnoise so this possibility will not be considered here although eventually it may replace the photomultiplier.

5.3.4. Increased post-detector bandwidth

Given very wideband photomultipliers the main technical problem in very wide post-detector bandwidths is the design of the linear multiplier. This could be eased, in principle, by a system using several video channels each with its own linear multiplier† but in any case an indefinite increase in post-detector bandwidth would impose quite unacceptable demands on the accuracy with which the time delays must be equalized around the two interferometer channels. In the Narrabri instrument with a bandwidth of 10^8 Hz the allowable differential time delay is 1 nanosecond and one would certainly not wish to decrease this by more than a factor of ten. No matter what improvements in photomultiplier bandwidth become available in the future an increase in sensitivity by one stellar magnitude seems to be the most that can be hoped for by this means.

5.3.5. Multi-channel observation

In an intensity interferometer only a very small fraction of the incident light is used in the interferometer measurement proper, since in principle the optical bandwidth need only be a few times the post-detector bandwidth. As well as leaving plenty of light for servo-controlled guidance of the mirrors this also gives one the possibility of simultaneous working with several channels each of which unfortunately would have to have its own pair of photomultipliers, linear multiplier and data handling equipment. The cost of the auxiliary equipment is likely to be heavy and one would be reluctant to aim for more than 16 channels, even in the favourable case of a single focus system which would give an increase in sensitivity of just over 1.5 stellar magnitudes. This could be achieved with eight different frequency bands in each of the two orthogonal states of linear polarization. Optically one could do this with a wide band polarizing beam splitter, two prism trains for light separation and background filtering, and a set of 100 Å wide interference filters as discussed below.

5.3.6. Improvements in the loss factors

If one looks at equation (5.2), where Σ was defined, and at the numerical values of the Narrabri equipment given in equation (5.7) one sees that the combined effect of loss of correlation and excess electronic noise contributed a factor 0.82 to Σ , which without excessive trouble one could hope to improve to 0.96. As we have already argued κ , the excess photomultiplier noise factor, could be reduced to 1.05 by use of the new high gain first dynodes. This leaves the problem of improving on the value of 0.42 for $\gamma(\nu_0)$, the optical efficiency of the interferometer. This is very much lower than this estimate of 0.7 given [3] in III and this loss of sensitivity of half a magnitude is the main reason for the fact that the Narrabri interferometer was limited in its initial measurements to stars of magnitude +2.0 rather than +2.5 as originally envisaged [11]. There were good practical reasons for this extra loss but these are not insuperable, at least with improved mirror accuracy which, among other things, would allow one to use a prism

† Ideally some of these problems could be greatly simplified if one were to use a coincidence counter in place of a linear multiplier but the arguments against this given in II still seem overwhelming.

train and thus eliminate the need for blocking filters†, and by taking every possible chance of improvement such as anti-reflection coating of all transmission optics and optimized interference filter characteristics one can hope that the 0.5 magnitude loss can eventually be recovered giving an overall improvement in Σ of nearly 1.25 magnitudes if we include the possible improvement in photomultiplier excess noise factor.

5.3.7. Overall possible improvement

In the table we have listed all the possible improvements in sensitivity. If all of these could be realized at once an overall improvement in sensitivity of 8.0 magnitudes could be made so that observation could be extended to stars with $m_{pg} = +10$. In fact, of course, one possible improvement is not really compatible with another. Thus multi-channel operation does not go well with multiple mirrors or with very wide-band operation because of the cost and complexity.

Technical change	Sensitivity improvement in stellar magnitudes
Increase in mirror size	3.25
Increase in post-detector bandwidth	1.00
Multi-channel operation	1.50
Quantum efficiency of detector	0.5
Improvement in loss factor	1.25
Simultaneous observation with short and long baselines	0.50
Total maximum possible improvement	8.0 magnitudes

Similarly the full improvement in Σ would be difficult with very large mirrors because of the excessive angular multiplication. However, if we throw away a factor of 2.5 magnitudes, that is a factor of 10, by using fewer channels, smaller mirrors and narrower bandwidths one would still be able to reach +7.5 magnitude stars. This conclusion is in agreement with the estimate published by Hanbury Brown [12] who argues that the Narrabri instrument could eventually reach stars in the range +4.0 to +5.0 magnitudes.

In order to work down to dim stars it is necessary that the effects of the night sky light and of correlated Cerenkov light pulses be not significant. We shall now consider what conditions this imposes upon the mirror quality.

5.4. Effect of night sky and of cosmic ray Cerenkov pulses

The night sky background at a good site is roughly equivalent to one 4th magnitude star per square degree at photographic wavelengths. If this background is to be less than 1 per cent of the light from a star of magnitude +7.5, this

† Given perfect optics it would seem that a prism train in polarized light incident at the Brewster angle would be the ideal way to obtain square shaped low loss filters. Unfortunately this would impose quite unrealistic limits in the design of the main mirror in view of the inevitably huge magnification factor between mirror aperture and filter optics. Indeed it is all one can do to reduce the angular spread of the light to the level where one can use a prism at all, even as a coarse prefilter, and even with the narrow field of view needed, in any case, to keep the night sky light to an acceptable level.

implies an acceptance cone for the mirror with an angle of a little more than 1' of arc and certainly this angle must be less than 3' of arc.

In III we considered [3] the possibility that Cerenkov radiation from cosmic ray showers could produce correlated light pulses in the two mirrors of the interferometer. Under extremely pessimistic conditions it was argued that such an effect might contribute 1 per cent of the correlation signal in observation of stars of magnitude +3.2 if the mirror acceptance cone was 12' of arc and if the optical passband was 100 Å. With a cone angle of 1' of arc, as required by the need to minimize the night sky light, this critical magnitude would rise to about +6. However, the treatment given [3] in III ignored the fact that the Cerenkov light from a shower does not emerge in a unique direction but is spread over a cone of solid angle W_c , where the cone angle is of the order 100' of arc. When one allows for this it is easy to show that the Cerenkov correlation effect is reduced by a further factor Ω_0/ω_c , where Ω_0 is the solid angle of the field of view of the mirror. This factor is of the order of 10^{-4} for a 1' of arc mirror so even with stars of magnitude +11 one would not expect the Cerenkov contribution to the correlation to be more than 1 per cent of that due to the star itself. This effect can therefore be totally neglected in optical intensity interferometry, a conclusion supported by the observational data from the Narrabri instrument.

5.5. Lower limit to stellar temperature

As was shown [2] in II the signal to noise ratio that is achievable on a source of given brightness temperature with zero baseline tends asymptotically to a finite limit as the mirror area is increased indefinitely. The reason for this was given in section 2 above, where we showed that for large mirrors which resolve the source the signal amplitude in the detector output increases only as the linear dimension of the aperture. The output of the linear multiplier therefore only increases linearly with the aperture area as does the shot noise power which is linearly proportional to the total light flux. The signal to noise ratio therefore ceases to increase with mirror aperture when the source is fully resolved.

Specifically if $Q(\nu_0)$ be the number of quanta of both polarizations emitted from unit area of the source in unit solid angle in unit time then this limiting value of the signal to noise ratio can be expressed in terms of $Q(\nu_0)$ by

$$\frac{S}{N}(T_0)_{\text{lim}} = \frac{\lambda_0^2 Q(\nu_0)}{2} \alpha(\nu_0) \rho(\nu_0)^{\text{sec } z} \left(\frac{b_\nu T_0}{2\eta} \right)^{1/2} \Sigma, \quad (5.9)$$

where the additional factor $\frac{1}{2}$ allows for the fact that the incident light beam has to be split in two in the case of zero baseline.

In terms of the brightness temperature $\Theta(\nu_0)$ of the source we have that

$$Q(\nu_0) = \frac{2}{\lambda_0^2} \left[\exp \left(\frac{b_\nu}{\kappa \Theta(\nu_0)} \right) - 1 \right]^{-1}, \quad (5.10)$$

so that

$$\frac{S}{N}(T_0)_{\text{lim}} = \frac{\alpha(\nu_0) \rho(\nu_0)^{\text{sec } z} \Sigma}{\left[\exp \left(\frac{b_\nu}{\kappa \Theta(\nu_0)} \right) - 1 \right]} \sqrt{\left(\frac{b_\nu T_0}{2\eta} \right)}. \quad (5.11)$$

With the parameters of the Narrabri inteferometer the coolest star that could be observed with the single mirror technique at $\lambda = 4430 \text{ \AA}$ has a brightness temperature of 4200°K corresponding approximately to a main sequence star of type K. With all the improvements in sensitivity discussed in the last section it would ideally be possible to reduce this minimum temperature to about 2900°K corresponding approximately to a main sequence star of type M. Actually if one were seriously concerned to measure cool stars with an intensity interferometer one would work not in the blue but at the longest wavelength at which adequate quantum efficiency could be obtained. However, it is for measurements on hot stars, which need very high resolving power, that the intensity interferometer is best suited and it is predominantly with these that we shall be concerned in what follows. Here it remains only to decide the dividing line between hot and cool stars in the present context. This question was answered in III by the criterion that the signal to noise-ratio must be adequate for observations with the baseline $d = D$, where D , the mirror diameter, is the smallest value of d possible with an interferometer formed by two separate mirrors. However, the experience with the Narrabri interferometer shows that this is not enough because with these limited baselines and with the partial coherence factor less than unity one would have considerable trouble in deciding whether the star was double. As long as there is no separate means of settling this it is necessary to follow the practice used at Narrabri and restrict observations to stars for which Δ is greater than a certain minimum which must surely not be less than 0.8. On this basis and assuming that zero baseline measurements with a split mirror were also possible the coolest stars that could be observed with the Narrabri interferometer using both mirrors would be of brightness temperature 4500°K , with the sensitivity increase of the last section this would come down to 3100°K .

These are minimum figures. If we define a 'hot' star to be one for which one can get ten times this sensitivity, corresponding to a value for Δ of 0.98 at the observational limit, these brightness temperatures rise to 6500°K corresponding to spectral type F 5 for the Narrabri interferometer and to 3850°K corresponding approximately to stars of spectral class K 5 if all possible improvements in sensitivity could be made.

These limiting values for the partial coherence factor $\Delta(\nu_0)$ apply of course to measurements at the limit of sensitivity. In brighter stars of the same spectral class one could either accept to work with a smaller value of $\Delta(\nu_0)$ as long as this was always greater than 0.8 or, alternatively, one could mask off, or otherwise limit, the mirror aperture.

5.6. *Measurements on hot single stars*

With an interferometer the basic information that one obtains is essentially a geometrical knowledge of the source since it is the angular distribution of apparent intensity that one measures. To determine the physical properties of the source, such as brightness temperature and effective black-body temperature one has to make subsidiary measurements and, possibly, invoke the help of theory, so these are inherently derivative not fundamental measurements. To begin with we will only consider the latter.

At minimum sensitivity in observations on an isolated hot star, an intensity interferometer measures one quantity and one quantity only, the angular diameter

of the equivalent uniform circular disc. If one is to reach stars of magnitude +7.5 one would need to go to baselines of between 2–4 km to resolve the very hot stars, a spacing comparable with that already used with large movable aerials in accurate radio astronomical interferometry.

At much higher sensitivity it should be possible to measure the limb darkening law directly. As shown in III one must do this by measuring the amplitude of the second maximum in the curve of 'fringe visibility' against baseline which requires a resolution about twice that needed to determine the angular width of the star. Unfortunately, because the intensity interferometer measures the square of the fringe visibility, this secondary maximum is small and varies from 0.017 for zero to 0.007 for full limb darkening. To measure this quantity with significant accuracy the star must be at least four magnitudes brighter than the dimmest star for which the angular size can be measured. If this be of magnitude +7.5 one would be restricted at best to stars brighter than +3.5. Furthermore, these must be hot enough to ensure that $\Delta(\nu_0)$ is not less than about 0.9 or the amplitude of the secondary maximum will be reduced by the resolution of the individual mirror. With the parameters of the Narrabri equipment this implies a brightness temperature greater than 9000°K which can be reduced to 4600°K by the improvements in sensitivity discussed in §5.3.

If one knows both the limb darkening law and the diameter of the equivalent uniform disc, one can find the angular diameter of the actual star. If one cannot measure the limb darkening one must rely upon theory supplemented with data from eclipsing variables. Fortunately as pointed out [3] in III the correction to the stellar diameter allowing for limb darkening is small, and the error will not exceed 5–6 per cent in the worst case, since the apparent variation in angular size for a star of given radius is 11 per cent between zero and full limb darkening. The limb darkening law is likely to vary with wavelength and this could lead, at the most sensitive choice of baseline, to a change in $\Gamma^2(d, \psi_0)$ of about 0.09 between full and zero limb darkening. Other things being equal this suggests that one could look for this effect over a frequency range for which the quantum efficiency was down by nearly a factor of ten on the peak value at which the limb darkening law could be measured directly. In practice of course things are not equal. With the cooler stars one could push more to the long wave end of the spectrum, with the hotter more to the short wave end where the quantum efficiency of modern photocathodes tends to be high anyway. In particular one could hope to measure the difference in angular size on either side of the Balmer discontinuity.

All this discussion was based on the assumption that the limb darkening law was approximately parabolic. In the super giant case it is possible that the model is closer to a hot central source surrounded by a much larger cooler envelope. In this case one probably has to make a detailed measurement of the shape of the 'fringe visibility' against baseline. This is likely to be quite time-consuming and could only be done on very bright stars. For the very hot O and B stars, which may be surrounded by an electron scattering atmosphere, one could expect the fringe visibility to be different in the two independent linear polarizations, one parallel the other perpendicular to the baseline, as has been pointed out by Kopal [13]. This effect might also be detectable on the very bright hot stars for which a very high signal to noise ratio could be obtained.

Finally, as pointed out by Hanbury Brown *et al.* [10], the apparent angular size of rapidly rotating stars might change by as much as 8 per cent, when the star

is seen equator on, depending on whether the rotation axis was parallel or perpendicular to the baseline. Such an effect could be studied quantitatively on stars some 3 magnitudes brighter than minimum.

By itself of course a mere knowledge of the angular size of a source is of little interest, because it does not say anything about the star itself. However, if the distance is known also one can then measure the actual radius of the star. If one knows the interstellar absorption one can deduce immediately the brightness temperature of the star, but to find the effective radiation temperature one also has to know the flux distribution over the whole spectrum. For the hotter stars this can only be found experimentally from satellite observations; until this data is to hand one must rely on theoretical models.

All this of course has been discussed before [3]. Given an interferometer technique of adequate sensibility clearly one would aim to establish a definitive temperature scale not only for different spectral and luminosity classes but also allowing for rotation, for metallic composition and, therefore, for age effects. However to do this one would hardly need to reach down to $m_{pg} = +7.5$ except perhaps to cover enough O and supergiant stars. There are about 14 000 stars in the sky brighter than this limit and over 90 per cent of them would be measurable with an intensity interferometer sensitive enough to reach $m_{pg} = +7.5$. Nevertheless there are good reasons for pushing the observational limit as far as possible. In the first place there are classes of stars of great special interest with few if any very bright members. Conspicuous among these are the classical Cepheids of particular importance because of their use as distance indicators. Only about a dozen could be measured at minimum as well as at maximum brightness with a limiting m_{pg} of $+7.5$ †. There are also about a dozen Wolf Rayet stars brighter than $m_{pg} = +7.5$ and rather more of the interesting emission stars of P Cygni and α Cygni type [14], and there are other classes of star which might merit special study. Moreover, at least as interesting as measurements of special stars are observations on those known to be members of open clusters such as the Pleiades. Many clusters are of quite small size, the distance spread of their members is small and as long as the distance of at least one member is known, interferometric measurements will give accurate measurements of stellar radii for all the other bright members. However, to do this over a reasonable range of spectral classes one must be able to work at least to $m_{pg} = +7.5$ since even in the Pleiades, one of the closest and most studied clusters the A stars are already at this limit.

Another benefit of being able to observe distant and therefore faint stars with an intensity interferometer is that the observations can be used to give a direct measurement of interstellar absorption as was first pointed out by Öhman [15] and more recently by Gray [16]. At present the total absorption in the blue is taken as four times the selective absorption, that is to say one assumes:

$$A_B = 4(A_B - A_V), \quad (5.13)$$

where $A_B - A_V$ is found from the apparent reddening, but it is certainly desirable to check this result by direct observation to see whether there is a component

† It is important to be able to observe the Cepheid over a complete cycle, that is at minimum as well as at maximum brightness, because one can then, in principle, combine the angular measurements with the velocity variations found spectroscopically to establish directly the distance and absolute luminosity of these stars.

of neutral absorption and whether this ratio varies over the galactic plane. To do this one has to find a class of hot stars of high luminosity with a well-defined brightness temperature as measured from nearby stars with small or known absorption. One could then use angular size measurements at large distances accurately to determine the absorption. Ideally one would like to use the hot supergiants of class Ia with absolute photographic magnitudes as high as -7.5 . In the absence of interstellar absorption such stars could be measured at 10 kiloparsecs by an interferometer limited to stars brighter than $m_{pg} = +7.5$ and even with an absorption of 2.5 mag/kpc one could reach distances up to 1.6 kiloparsecs. An accurate knowledge of the absorption would also improve the accuracy of spectroscopic parallaxes especially for the hot stars at large distances.

5.7. Measurements on double and multiple hot stars

Let us consider the case of a multiple star, the components of which are each of an angular size so small that they are not significantly resolved by the individual mirrors of the intensity interferometer. In this case $\Gamma^2(d, \psi_0)$, the 'fringe visibility' measured by the interferometer, is given by:

$$\left(\sum_{r=1}^N I_r\right)^2 \Gamma^2(d, \psi_0) = \sum_{r=1}^N I_r^2 \Gamma_r^2(d, \psi_0) + 2 \sum_{r>s}^N \sum_{s=1}^{N-1} I_r I_s \Gamma_r(d, \psi_0) \Gamma_s(d, \psi_0) C_{rs}^2(\theta_{rs}, D) \cos\left(\frac{2\pi\theta_{rs}d \cos \overline{\psi_{rs} - \psi_0}}{\lambda_0}\right), \quad (5.14)$$

where

I_r is the intensity of the r th component,

$\Gamma_r(d, \psi_0)$ is the Fourier transform of the light intensity over the equivalent line source of the r th component projected onto a baseline of orientation ψ_0 ,

θ_{rs} is the angular separation of the r th and s th components,

$\psi_{rs} - \psi_0$ is the angle between the line joining the r th and s th components and the baseline of orientation ψ_0 ,

$C_{rs}(\theta_{rs}, D)$ is the factor expressing the partial resolution of the double star formed by the r th and s th components by a single mirror of the interferometer.

For a circular mirror diameter D :

$$C_{rs}(\theta_{rs}, D) = \frac{2J_1(\pi\theta_{rs}D/\lambda_0)}{\pi\theta_{rs}D/\lambda_0}. \quad (5.15)$$

As long as the individual stars are circularly symmetrical, that is as long as $\Gamma_r^2(d, \psi_0)$ is independent of the baseline orientation ψ_0 , it follows from equation (5.14) that the intensity interferometer can be used to find not only $\Gamma_r^2(d, \psi_0)$ for each component, but also the angles θ_{rs} and ψ_{rs} subject only to certain mirror type ambiguities in the orientation of the system. Of course if there are a number of components of comparable brightness a large number of accurate observations at different baseline orientations would be needed to unravel the picture but fortunately although multiple stars are not at all uncommon there is usually a

large difference in the luminosities when the dimmer components can effectively be ignored in interpreting the interferometer data.

Accordingly, in what follows, we shall consider the simple case of a double star, for which there are two extreme cases of special interest:

- (1) $C_{12}(\theta_{12}, D) \approx 0$,
- (2) $C_{12}(\theta_{12}, D) \approx 1$.

In the first case there can be no question of measuring the angular separation of the components. All one would know is that the correlation at zero baseline is reduced below that expected from a point source of the same apparent luminosity by the ratio†:

$$\Delta = \frac{\sum_{r=1}^N I_r^2}{\left(\sum_{r=1}^N I_r\right)^2}. \quad (5.1)$$

When Δ is close to unity we can assume that

$$\sum_{r=2}^N I_r^2 \ll I_1^2,$$

when the interferometer will only measure the intensity distribution over the bright component. When evaluating the brightness temperature in this case it is necessary only to note that the flux from the bright component is a fraction

$$\frac{2\Delta}{1+\Delta}$$

of the total received flux.

However, when Δ is not close to unity the picture is very confused, one would not even know how many components were being measured by the interferometer, so this case must be excluded.

The second alternative when $C_{12}(\theta_{12}, D)$ is close to unity is more important for astronomical purposes because the star is then very likely to be a spectroscopic doublet. If it is, then the interferometer enables one to measure those parameters, such as the inclination i and the orientation of the orbit with respect to the galactic plane, which are not given by the spectroscopic data. If both sets of lines are visible this means that one can get accurate measurements of stellar masses on stars other than eclipsing variables, because i is known, and one can also obtain the distance‡ of the star. This would be particularly valuable if the star were a

† To distinguish the presence of a fully resolved multiple star from a large feature, such as an extended atmosphere, of a single star one only has to take zero baseline measurements with two mirror apertures. In the former case the partial coherence factor would vary, in the latter it would not.

‡ This is straightforward when the two components are so far apart that they radiate like uniform circular discs. For very close stars with ellipsoidal components the problem is more difficult. However, in this case the star is very likely to be an eclipsing variable. One would need the photometric eclipse data to derive a stellar model from which the Fourier transform of the light distribution could be found with two free parameters, the scale in angular measure and the orientation to the baseline, which would be determined by the interferometer data.

member of a cluster, as this would give an independent and accurate measure of the cluster distance.

For both sets of spectroscopic lines to be visible it is necessary that the two components have approximately the same magnitudes. However even when the components differ by four magnitudes $\Gamma^2(d, \psi_0)$ will still vary by some 10 per cent with baseline orientation ψ_0 . Such a star could be measured with an interferometer as long as its magnitude was some 2.5 magnitudes brighter than the +7.5 limit and one could still find the stellar masses and radii if the distance were known. Even if no trigonometrical parallax were available the star might belong to a cluster of known distance or an accurate spectroscopic parallax might exist for the bright component.

The attraction of using an interferometer to get masses of spectroscopic binaries is firstly that there are many more of such stars than there are of eclipsing variables the only source of information so far on the masses of hot stars $M_{pg} < 0$. Secondly, these eclipsing stars are usually extremely close binaries not necessarily typical of the stellar population as a whole.

One would still want to observe eclipsing stars, however, firstly to get the distances and orbit orientation, but also because they provide an excellent test of the reliability of interferometer measurements. A good example in the Northern Hemisphere is provided by β Aurigae, a pair of Ao V stars, as has been pointed out by Kopal [17].

To find the conditions under which $C_{12}(\theta_{12}, D) \approx 1$ we may note that the maximum angular separation of a spectroscopic binary at a distance of p parsecs is given by:

$$\theta_0 = \frac{(M_1 + M_2)^{1/3} P^{2/3}}{51p}, \quad (5.17)$$

where

θ_0 is in seconds of arc,

P is the period in days,

M_1, M_2 are the stellar masses expressed in units of solar mass.

On the same basis the minimum angular separation will be $\theta_0 \sin i$, where i is the inclination of the orbit.

If $C_{12}(\theta_0, D)$ is to be greater than 0.88 it is necessary that

$$\frac{\pi \theta_0 D}{\lambda} \gtrsim 1. \quad (5.18)$$

With a mirror diameter in the range 20–30 m needed to reach stars with $m_{pg} \geq +7.5$ and with observations in the blue, this implies, for binaries observed near minimum sensitivity, that

$$\theta_0 \gtrsim 10^{-3} \text{ sec of arc.} \quad (5.19)$$

Assuming that the binary components obey the mass-luminosity law it can be shown that the maximum period for which the orbital separation obeys the inequalities (5.19) for a star measured near the threshold magnitude of $m_{pg} = +7.5$ is given approximately by:

$$P < 15 \times 10^{-0.24 M_{pg}} \text{ days,} \quad (5.20)$$

where M_{pg} is the absolute photographic magnitude of the binary. In deriving 5.20 the effects of interstellar absorption were neglected and the mass of an A o star was taken [18] as 3.55 solar masses with $M_{pg}=0.7$.

The constraint 5.20 would not affect many spectroscopic binaries with both sets of lines visible and of spectral class earlier than A o, but it would seriously limit the number of double stars of type F and later that could be successfully observed with an intensity interferometer.

For nearby stars with periods close to the critical limit set by the inequality 5.20 it would be necessary to limit the mirror aperture, but the resulting loss of signal to noise ratio will just be balanced by the increase in apparent brightness so the criterion of 5.20 applies irrespective of apparent magnitude as long as this latter is above threshold.

6. The laser in spatial intensity interferometry

The idea of using a laser in spatial intensity interferometry was first suggested by Gamo [10] chiefly on the grounds that this would enable one to measure the 'fringe visibility' rather than the square of this quantity. At first glance one would also hope for a greatly improved signal to noise ratio but, at least at optical wavelengths, where one can use photomultiplier quantum detectors this is not the case because of the enormous shot noise associated with the laser itself. Allowing for this it turns out that the only change in the signal to noise ratio as given by equation (5.1) is that the term $\Gamma^2(d, \psi_0)$ is replaced by $4\Gamma(d, \psi_0)$ while $\eta_0(\nu_0)$ the number of photons $m^{-2} s^{-1} (c/s)^{-1}$ now refers not to both polarizations but only to the polarization of the incident laser light. Effectively this means an improvement in signal to noise ratio of $2/\Gamma(d, \psi_0)$ which certainly would be well worth having if it came for free. Unfortunately the practical objections to using a laser for optical astronomical applications to spatial interferometry are overwhelming, [20] and since they apply with varying force to all other laser applications in the visible band it seems worth while to develop them here in more detail.

In the first place, if the laser is to be used to measure $\Gamma(d, \psi_0)$ as opposed to $\Gamma^2(d, \psi_0)$, it is being used exactly as a local oscillator would be at radio wavelengths with the photomultiplier acting as a linear frequency changer rather than as a square law detector. In this case one no longer has an intensity interferometer but a Michelson interferometer with all the problems in phase stability that that involves and with very little corresponding increase in sensitivity. Admittedly one could eliminate the laser phase, just as at radio wavelengths, by following the photomultiplier by a square law detector between the video amplifier and the linear multiplier but then we should still measure $\Gamma^2(d, \psi_0)$ while the effect upon the signal to noise ratio at optical wavelengths would be catastrophic. In any case once one introduces a laser local oscillator out goes the hope of using a crude mirror because the phase of the beat note between the laser and the signal from the star must be constant to within a fraction of a radian over the whole aperture if significant loss of correlation is to be avoided. More specifically the mirror surface must be good to something of the order of $\lambda/20$ while the pointing error $\Delta\theta$ must satisfy the inequality:

$$\Delta\theta = \frac{\lambda}{10D}, \quad (6.1)$$

where D is the mirror diameter. With $\lambda = 4400 \text{ \AA}$ and $D = 30 \text{ m}$, as needed to reach $m_{pg} = 7.5$, the inequality (6.1) implies a pointing accuracy better than $3 \times 10^{-4} \text{ sec of arc}$! Nor is this all, because atmospheric turbulence will introduce irregular phase variations, with an r.m.s. amplitude of at least several radians over a large mirror aperture, which cannot be corrected even by perfect mirror optics. Conceivably the 'seeing' effects as well as the imperfections of a crude mirror could be overcome, as implied in the discussion between Gamo and Radakrishnan [20], by placing the laser above the atmosphere on a satellite, but this is indeed the counsel of Munchhausen. In the first place if the laser light is to suffer the same 'seeing' deflections as the starlight it must take the same path through the atmosphere which implies that the star and the laser bearing satellite must be aligned to something of the order of 1 sec of arc at both mirrors. However, this constraint is trivial compared with that imposed by equation (6.1). With the 3000 m baseline needed to resolve the hottest stars of magnitude $m_{pg} = +7.5$ it is necessary that the laser circulate at a distance not less than $2 \times 10^9 \text{ km}$ from the Earth, which is more than 10 times the distance of the Sun, with an angular error of not more than $3 \times 10^{-4} \text{ sec of arc}$ and all for a factor of $2/\Gamma(d, \psi_0)$ in signal to noise ratio! Even with single mirror observations, the satellite must be more than 20 000 000 km from the Earth if the necessary phase alignment is to be maintained over the aperture of a 30 m mirror.

7. Intensity interferometry in scattered radiation

7.1. Incoherent scattering

Up to now we have only considered the case in which the object under study is itself an emitter of radiation which is incoherent as between one part of the object surface and another. Clearly the theory would also apply to a cold body illuminated with incoherent radiation, confined if desired to a narrow band, but a more interesting possibility is that in which the object is a random scatterer illuminated with coherent radiation, possibly pulsed, when one could still use the intensity interferometer principle to determine the angular size. Ideally a random scatterer in this context is one where there is zero correlation between the phase of the radiation scattered from one element of the target and another. This can never be realized perfectly in practice but a rough body might be a good enough approximation as long as the irregularities were large compared with the wavelength and as long as the scattering pattern was reasonably isotropic.

7.2. Coherent scattering

Of more fundamental interest than a rough target is the possibility of observations with radiation that has been scattered coherently from a regular target such as a crystal lattice. It has been suggested by Goldberger *et al.* [21] in a long and detailed discussion that one could use the intensity interferometer principle to measure the relative phase of x-ray amplitudes scattered by a regular crystal. One case considered by these authors was that in which a target near the origin was illuminated by *two* sources of radiation at vector positions $\mathbf{l}$ and $\mathbf{\lambda}$, the scattered radiation being observed by two detectors at vector positions $\mathbf{a}$ and $\mathbf{\alpha}$. As long as the two sources appear optically superimposed at *both* detectors

positions then it was shown that, in principle, an intensity interferometer could measure the quantity,

$$\cos [\phi(\mathbf{l}, \mathbf{a}) - \phi(\boldsymbol{\lambda}, \mathbf{a}) - \phi(\mathbf{l}, \boldsymbol{\alpha}) + \phi(\boldsymbol{\lambda}, \boldsymbol{\alpha})],$$

where $\phi(\mathbf{l}, \boldsymbol{\alpha})$ is the phase of the scattering amplitude due to radiation from the source at $\mathbf{l}$ reaching the detector at $\mathbf{a}$.

One trouble with this, pointed out by the authors [21], is that it is in general only possible to achieve optical coincidence at a given frequency which is itself a function of scattering angle. Much more serious however is the problem, always present in intensity interferometry, of getting adequate sensitivity. Goldberger *et al.* state that the intensity difficulties are overcome by using irradiated targets rather than natural emitters but in the view of the present writer this statement is completely invalid. On the contrary because the source radiation is split up into a number of beams by the crystal target the signal to noise ratio is always lower than when one views the target directly. One could prove the truth of this with a general mathematical argument but we prefer here to discuss a simple special case in enough detail to make clear that the argument would apply with equal force to the general scattering problem.

Consider the case where the 'scattering' crystal consists of an ideal 50–50 beam splitter, with zero reflection from the second face, illuminated by two sources and observed with two detectors. In this case at least if the two sources appear in optical coincidence in one beam they are so also in the other. To begin with let us assume that the sources are very small, of equal intensity I_0 , and either coherent but different frequency transmitters, as considered in § 2, or hot thermal emitters. If θ_0 be the apparent angular separation of the two sources as viewed from either detector then the beat note in the detector output due to interaction between the sources will be negligible as long as

$$\frac{\theta_0 D}{\lambda_0} \gg 1,$$

where λ_0 is the mid-band wavelength and D is the width of the detector. The signal output of the intensity interferometer will be proportional to

$$\frac{I_1^2 + I_2^2}{4} = \frac{I_0^2}{2}$$

for the case of two thermal sources and will be zero for the case of two monochromatic transmitters. As the sources come into optical coincidence however, that is as $\theta_0 D / \lambda_0 \rightarrow 0$, the interaction between the superimposed sources will make a *negative* contribution to the output of the intensity interferometer because of the relative phase change of π in the radiation reflected from one side of the beam splitter to that reflected by the other. In the case of the thermal radiators the intensity interferometer output would fall to zero at optical coincidence, for the case of two monochromatic transmitters a negative output proportional to $-I_0^2/2$ would be obtained. Now consider what happens when these sources are thermal and of finite angular size $\bar{\theta}$. The total amplitude of the beat note due to interaction between two different parts of the same source will fall off as soon as $\bar{\theta} D / \lambda_0$ is no longer small compared with unity because the phase of the beat note

varies over the detector surface, and exactly the same effect will arise due to interaction between one part of one source and another part of the superimposed source. In both cases one is limited by the effective brightness temperature of the source and with incoherent x-ray sources there is absolutely no prospect that the sensitivity would be adequate for any scattering phase measurement. It is possible that x-ray lasers will one day become available, the difference in the phase of the beat notes in the two detectors at $\mathbf{a}$ and $\mathbf{\alpha}$ due to interaction between the two laser frequencies is then related as we have seen to the relative phase differences of the scattering amplitudes and it is just conceivable that one could thus obtain a direct measurement of these, but this is a very long shot.

8. The intensity interferometer in frequency measurement

The possibilities of such measurements have been discussed in some detail elsewhere [22], where it was shown that their value is extremely limited. They certainly have no possible applications in visual astronomy because among other things one is never dealing in stellar spectra with the case of one or two isolated intense very narrow emission lines, the only case suitable for very high resolution work with a temporal intensity interferometer. Admittedly, things are more promising if one can use a laser because it is then much easier to sort out a complex spectrum. However this has no application in visual astronomy for the reasons outlined in §6. Some laboratory applications might be envisaged in studying very hot emission lines but even these involve long integration times and accurate alignment, and the possible applications are further reduced by the fact that the signal to noise ratio is inevitably well below the maximum obtainable in angular measurements because in high definition frequency measurement one has of necessity to work with comparatively narrow post-detector bandwidths.

9. Higher order intensity interferometry

The original measurements on the radio source Cygnus A with an intensity interferometer led to two possible models for the source, one with two, the other with three emission regions. In principle it is possible to distinguish between these by accurate measurements, at wider baselines, of the Fourier amplitudes alone but a more reliable and sensitive method is to measure the difference in the phase of the Fourier transform with baselines chosen to lie on either side of the first minimum in the Fourier amplitude curve. This phase difference is π radians for one model and zero for the other.

As is well known, this information cannot be obtained with a two-aerial intensity interferometer and Jennison eventually established the validity of the model with two emission regions using *three* aerials simultaneously and an ingenious modification of this Michelson interferometer [23, 24] which enabled him to measure:

$$\Delta\Phi(d_1, d_2) = \Phi(d_1 + d_2) - \Phi(d_1) - \Phi(d_2),$$

where $\Phi(x)$ is the Fourier phase appropriate to baseline x , and where d_1, d_2 are the baseline separations of the two outer aerials from the centre.

It was later pointed out by Gamo [25], in a discussion of frequency spectrum measurements in the optical case, that there is an intensity interferometer equivalent to Jennison's interferometer which also permits one to measure $\Delta\Phi(d_1, d_2)$.

In the modification one also makes simultaneous observations with *three* aerials and multiplies the low frequency a.c. outputs of the three associated square law detectors in a triple correlator. At the same time one multiplies the video outputs of the detectors in pairs to obtain the Fourier amplitude. Combining all this data enables one in principle to measure:

$$\cos [\Phi(d_1 + d_2) - \Phi(d_1) - \Phi(d_2)].$$

Unfortunately, as one would expect, this higher order multiplication leads to a further loss of signal to noise ratio by a factor proportional to

$$\left(\frac{\alpha n_0 b_v}{B}\right)^{1/2} \Gamma(d_1 + d_2)$$

in the optical and to

$$\frac{P_s}{P_N + P_s} \Gamma(d_1 + d_2) \sqrt{\left(\frac{b_v}{B}\right)}$$

in the radio case, where the symbols have the same meaning as in §5 above and where we have made the assumption $b_v \leq B$ as was done in deriving equations (5.1) and (4.1) respectively.

From this it follows at once that optical applications of this technique to astronomy can be quite ruled out because $\alpha n_0 \sim 10^{-4}$, $b_v/B \sim 10^{-4}$ in a typical case. For the radio case, however, things are less desperate because one can now match the post-detector and pre-detector bandwidths for optimum sensitivity and for strong sources observed with large aerials, P_s/P_N may not be much less than unity. Indeed had the possibility of the triple intensity interferometer been realized earlier it is possible that it would have been used to settle the ambiguity of the Cygnus A radio model. If so it would have been this first and so for the only *application* of higher order photon correlation†.

Anhand eines einfachen Beispiels werden Merkmale für die Beantwortung der Frage aufgestellt, wann ein Intensitäts-Interferometer mit einem Michelson-Interferometer konkurrieren kann. Auf dem Gebiet der Frequenzmessung scheint dies in keinem Falle zuzutreffen. Es wird zwar gesagt, dass bei der Radioastronomie die Intensitäts-Interferometrie doch noch bei der Winkelmessung sehr kleiner Strahlungsquellen eine Rolle spielen, aber es liegt auf der Hand, dass die Hauptanwendung in der visuellen Astronomie bei den Messungen an heißen Einzelsternen und benachbarten spektroskopischen Doppellinien liegt. Es wird erörtert, in wie weit diese Methode verbessert werden kann, einmal von dem technischen, dann aber auch vom Standpunkt der Beobachtung aus. Dann wird auch das Interferometer mit dreifacher Korrelation nach Gamo behandelt; aber wegen seiner ausserordentlich geringen Empfindlichkeit scheint es keine Verwendung gefunden zu haben, wenigstens nicht in der visuellen Astronomie. Auf diesem Gebiet scheiden auch die Laser aus, teils wegen der verschiedenartigen Einflüsse atmosphärischer Störungen, teils wegen der überaus engen Toleranzen für die Ausführung des Spiegels, der für eine ausreichende Empfindlichkeit sehr gross werden muss. Die Anwendungsmöglichkeiten im Laboratorium scheinen auch beschränkt zu sein; insbesondere scheitert der Gedanke, die Intensitäts-Interferometrie zur Messung der Phase in Streuamplituden im Bereich der Röntgenwellen zu messen, an der unzureichenden Empfindlichkeit, die man nur durch die Entwicklung kohärenter Röntgenstrahlquellen steigern könnte.

† Higher order correlations have been studied in measurements of laser photon statistics but no applications have yet emerged.

REFERENCES

- [1] HANBURY BROWN, R., and TWISS, R. Q., 1957, *Proc. R. Soc., A* **242**, 300.
- [2] HANBURY BROWN, R., and TWISS, R. Q., 1957, *Proc. R. Soc. A*, **243**, 291.
- [3] HANBURY BROWN, R., and TWISS, R. Q., 1958, *Proc. R. Soc. A*, **248**, 199.
- [4] HANBURY BROWN, R., and TWISS, R. Q., 1958, *Proc. R. Soc. A*, **248**, 222.
- [5] HANBURY BROWN, R., JENNISON, R. C., and DAS GUPTA, M. K., 1952, *Nature, Lond.*, **170**, 1061.
- [6] JENNISON, R. C., and DAS GUPTA, M. K., 1953, *Nature, Lond.*, **72**, 996.
- [7] GUBBEY, J. S., and ROBERTSON, D. S., 1967, *Nature, Lond.*, **215**, 1157.
- [8] KELLERMAN, K. I., *et al.*, 1968, *Ap. J. Letters*, **153**, L209.
- [9] MICHELSON, A. A., and PEASE, F. G., 1921, *Astrophys. J.* **53**, 249.
- [10] HANBURY BROWN, R., DAVIS, J., ALLEN, L. R., and ROME, J. M., 1967, *Mon. Not. R. astro. Soc.*, **137**, 393.
- [11] HANBURY BROWN, R., DAVIS, J., and ALLEN, L. R., 1967, *Mon. Not. R. astr. Soc.*, **137**, 375.
- [12] HANBURY BROWN, R., 1968, *A. Rev. Astronomy Astrophys.*, **6**, 13.
- [13] KOPAL, Z. (private communication).
- [14] ALLEN, C. W., 1963, *Astrophysical Quantities*, 2nd edition (The Athlone Press), p. 216.
- [15] OHMAN, Y., (private communication).
- [16] GRAY, D. F., 1968, *Ap. J. Letters*, **153**, L113.
- [17] KOPAL, Z. (private communication).
- [18] ALLEN, C. W., *loc. cit.*, p.203.
- [19] GAMO, H., 1961, *Advances in Quantum Electronics*, edited by J. R. Singer (New York: Columbia University Press), p.252.
- [20] RADAKRISHNAN, V., 1961, *Advances in Quantum Electronics*, edited by J. R. Singer (New York: Columbia University Press), p. 252.
- [21] GOLDBERGER, M. L., LEWIS, H. W., and WATSON, K. M., 1963, *Phys. Rev.*, **132**, 2764.
- [22] GEBBIE, H. A., and TWISS, R. Q., 1966, *Rep. Prog. Phys.*, 729.
- [23] JENNISON, R. C., 1958, *Mon. Not. R. astr. Soc.*, **118**, 276.
- [24] JENNISON, R. C., and LATHAM, V., 1959, *Mon. Not. R. astr. Soc.*, **19**, 174.
- [25] GAMO, H., 1963, *J. appl. Phys. (U.S.A)*, **34**, 875.